Nonlinear Coupled Dynamics of a Movable Capacitor Plate

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***Abstract* - This study focuses on analyzing the interaction between electrical and mechanical systems using a coupled capacitor-plate mechanism. The capacitor consists of aluminum plates with air as the dielectric medium, and its dynamic behavior directly influences the motion of a spring-connected movable plate. A stainless steel (302) spring is selected to model the mechanical restoring force, and system parameters are carefully chosen based on theoretical analysis.**

**The governing equations are formulated by combining the voltage divider rule for the electrical circuit and Newton's second law for the plate's motion. A nonlinear second-order differential equation describing the position and velocity of the plate is solved numerically using MATLAB’s ODE45 solver.**

**The results show the system's position and velocity oscillations under varying electrostatic forces. Additionally, the dynamic resistance (R1) is analyzed as a function of plate displacement, revealing the interplay between mechanical motion and electrical behavior.**

***Index Terms*—modeling and simulation, electromechanical system, nonlinear dynamics, capacitor-spring interaction, MATLAB, dynamic resistance.**

This report is the collective work of Uğur Can KIZILCAN (200029971) and Merve ÇABUK (200030470). Each author is responsible for their own discipline: Uğur Can KIZILCAN - Electromechanical system & Modelling and Simulation, Merve ÇABUK – Mechanical System.

**INTRODUCTION**

In this project, we aim to analyze and simulate a coupled electromechanical system where a movable capacitor plate interacts with a spring mechanism. The system comprises a parallel-plate capacitor with aluminum plates and air as the dielectric medium, chosen to simplify the design while ensuring accurate theoretical modeling. Unlike systems with fixed dielectrics such as PTFE, the use of air introduces dynamic changes in capacitance due to plate displacement.

The mechanical behavior of the system is governed by a spring made of stainless steel 302, selected for its high elasticity and durability, which provides a restoring force proportional to the displacement. The stiffness constant (k) is derived theoretically based on material properties and geometry.

The core objective of this study is to model the coupled dynamics of the system using fundamental electrical and mechanical principles:

1. The voltage divider rule determines the voltage across the capacitor.
2. Newton's second law of motion governs the mechanical displacement and oscillatory behavior of the plate under combined electrostatic and spring forces.

By solving the derived nonlinear motion equations using MATLAB's ODE45 solver, we aim to observe:

1. The oscillatory position and velocity of the movable plate over time.
2. The dynamic behavior of the variable resistance () as a function of plate displacement.

This study provides insight into the interplay between electrostatic forces and mechanical motion, which has applications in precision systems such as MEMS devices and capacitive sensors. The results are presented through graphical analysis of position, velocity, and resistance changes over time.

**SYSTEM COMPONENTS**

The electromechanical system analyzed in this project consists of two main components: an **electrical subsystem** and a **mechanical subsystem**. The interaction between these components drives the dynamics of the system, where electrostatic forces influence the motion of a spring-connected movable plate.

**A. Electrical Subsystem**

**1. Capacitor**

The capacitor is designed as a parallel-plate structure with:

- ***Plate Material***: Aluminum (chosen for its lightweight nature, high conductivity, and cost-effectiveness).

- ***Dielectric Medium:*** Air (instead of solid dielectrics like PTFE), which introduces dynamic changes in capacitance due to the varying distance between plates.

The capacitance of the system is given as:

where:

1. **C**: Capacitance (F),
2. = 8.85 x F/m: Vacuum permittivity,
3. **A** = 0.0009 : Plate area,
4. **d** = 46.48 x m: Initial plate separation,
5. **x:** Plate displacement over time.

The varying plate distance dynamically alters the capacitance, which in turn affects the electrostatic force acting on the plates.

**2. Voltage Divider**

The capacitor voltage ( Vc) is determined using the voltage divider rule across resistors (x) and :

where:

1. = 1 kΩ,
2. **(x)** = is a dynamic resistance that changes with plate displacement.
3. = 5V: Input voltage.

**B. Mechanical Subsystem**

**1. Spring**

Initially, we considered using the experimental formula to determine the spring constant **k =**

This formula calculates the spring constant based on the static elongation **(ΔL)** of the spring under a known weight **(m)** and gravitational force **(g**). It is derived from Hooke's Law **(F=k⋅ΔL)** and Newton's second law **(F=m⋅g).** While this method is simple and effective for experimental validation, it assumes a purely static system where forces are balanced, and no dynamic effects (e.g., oscillations) are considered.

However, since our project requires a more precise, theoretical approach to model the dynamic behavior of the system, we transitioned to using the following formula for the spring constant **(k)**:

This equation is derived from the material properties and geometric parameters of the spring:

1. **G:** Shear modulus of the material (stainless steel 302 in our case),
2. **d:** Wire diameter,
3. **D:** Mean coil diameter,
4. **N:** Number of coils.

By incorporating these parameters, we obtained a spring constant of approximately **k=7.292362e+02 N/m**, which aligns with the material properties and structural geometry of the selected spring. This theoretical value is better suited for our system, as it accounts for the mechanical design and ensures consistency with the overall model.

In summary, while the experimental formula(**k=** ​) provides a useful validation tool, we opted for the theoretical model to maintain accuracy and reliability in our dynamic system simulations.

**2. Newton’s Second Law of Motion**

The plate's motion is influenced by two main forces:

1. Restoring force from the spring: **Fs = −kx ,**
2. Electrostatic force from the capacitor:

Combining these forces, the motion of the plate is described using Newton's second law: **m+kx=Fc** where **m=33.75× kg** is the mass of the plate.

**MATHEMATICAL MODEL**

To model the dynamics of the system, we combined the governing equations of both the electrical and mechanical subsystems. The system’s behavior is represented by a second-order nonlinear differential equation, which is solved numerically.

**A. Electrical Subsystem**

The voltage across the capacitor **()** is calculated using the voltage divider rule:

where **(x) =**  represents the dynamic resistance. The electrostatic force **(Fc)** acting on the capacitor plates due to this voltage is given by:

Here, depends on the displacement **(x)** of the movable plate, making **Fc** a nonlinear function of x.

Additionally, the current flowing through the capacitor **()** is given by:

where:

1. ​ is the capacitance of the system,
2. **​** is the voltage across the capacitor.

By substituting C and , the current becomes:

This equation captures how the current through the capacitor changes with time, incorporating the effects of the plate displacement (x) and circuit parameters.

**B. Mechanical Subsystem**

The mechanical behavior of the system is governed by Newton’s second law of motion:

**m+kx=Fe where:**

1. **m:** Mass of the movable plate,
2. **k:** Spring constant,
3. **x:** Displacement of the plate,
4. **Fe:** Electrostatic force.

By substituting the expression for Fe, the governing equation becomes:

**m+kx=**

This nonlinear differential equation couples the electrical and mechanical dynamics. Solving this equation provides the position **(x)** and velocity **()** of the plate over time.

**C. Numerical Solution**

To solve the differential equation, MATLAB's ODE45 solver is used. The equation is rewritten as a first-order system by defining:

**= x** and  **=**  . The system can then be expressed as:

**, =**

The following initial conditions are used:

indicating that the plate starts from rest at its equilibrium position.

**D. Event Condition for Safety**

An event condition is implemented to ensure that the plate displacement **(x)** does not exceed a critical value **(−0.415× m)** which would short-circuit the capacitor: value = **- (−0.415× ),** with “isterminal **=1”** to stop the simulation when the displacement exceeds this limit.

**E. Simulation Output**

The simulation provides the following results:

1. ***Position vs. Time (x(t)):*** Shows the oscillatory behavior of the plate.
2. ***Velocity vs. Time ():*** Illustrates how the speed of the plate changes dynamically.
3. ***Resistance () vs. Displacement (x):*** Demonstrates the relationship between plate position and dynamic resistance.
4. ***Change of the Charge (Q) of the Capacitor:*** Shows the change of the capacitors charge over time.
5. ***Current Through the Capacitor:*** Shows the current flowing through the capacitor.
6. ***Current Through the R1 Resistor:*** Shows the current flowing through the R1 resistor.
7. ***Total Current of the System:*** Shows the current flowing through the R2 resistor and the total current flow of the system.

These outputs are used to analyze the coupled dynamics of the system.

ekran görüntüsü, metin, diyagram, tasarım içeren bir resim

Açıklama otomatik olarak oluşturuldu

**SIMULATION RESULTS**

The simulation results obtained from MATLAB reveal the dynamic behavior of the electromechanical system. The position, velocity, and dynamic resistance of the movable plate were analyzed over time.

**A. Position vs. Time**

The position of the movable plate (x(t)) exhibits an oscillatory behavior, as shown in the graph.

**Graph: Position vs. Time**

The plate initially moves due to the electrostatic force generated by the capacitor. The restoring force of the spring counteracts this motion, resulting in oscillations. The amplitude of these oscillations decreases over time due to damping effects, which were not explicitly modeled but are observed as a result of system energy dissipation.

1. The plate displacement remains within the physical limits **(−0.415× m)** to prevent capacitor short-circuiting.
2. The oscillatory nature indicates the coupling between the spring force and the electrostatic force.

**B. Velocity vs. Time**

The velocity of the plate **(t))** is shown in the second graph

**Graph: Velocity vs. Time**

As the plate oscillates, its velocity alternates between positive and negative values, corresponding to forward and backward motion. The maximum velocity occurs near the equilibrium position where the restoring force is minimal.

1. The velocity peaks align with the zero-crossings of the position graph, as expected in harmonic motion.
2. The velocity decreases over time, consistent with the damping-like behavior observed in the position graph.

**C. Resistance ((x)) vs. Displacement (x)**

The relationship between the dynamic resistance **(x)** and the displacement of the plate **(x)** is illustrated in the third graph:

**Graph: Resistance vs. Displacement**

The resistance **(x)** is inversely proportional to the displacement x, as defined by the equation **(x)** = ax+b. This relationship is crucial for understanding how the electrical behavior of the system adapts to mechanical motion.

1. As the plate moves closer to the other capacitor plate ***(x decreases), (x) increases*** sharply.
2. This behavior introduces nonlinearity in the system, which impacts the oscillatory response.

**D. Summary of Results**

1. The system exhibits coupled oscillatory motion driven by electrostatic and mechanical forces.
2. The dynamic behavior of **(x**) introduces nonlinearity, affecting the voltage across the capacitor and the resulting forces.
3. The safety condition **(x ≥ (−0.415× m )** is successfully maintained throughout the simulation, ensuring no short circuit occurs.

These results highlight the interplay between electrical and mechanical subsystems, showcasing how changes in one domain influence the other.

**CONCLUSION**

This project successfully analyzed the dynamic behavior of an electromechanical system where a parallel-plate capacitor interacts with a spring-connected movable plate. The system's nonlinear dynamics were modeled mathematically, and numerical solutions were obtained using ***MATLAB's ODE45*** solver.

1. **Oscillatory Motion:** The plate exhibits oscillatory behavior under the combined influence of electrostatic and spring forces. The amplitude and velocity of these oscillations decrease over time due to energy dissipation.
2. **Dynamic Capacitance and Resistance:** The varying capacitance and dynamic resistance **(x**) introduce nonlinearity into the system, significantly affecting the voltage across the capacitor and the resulting electrostatic forces.
3. **Safety Compliance:** The plate displacement remained within safe limits, ensuring that the capacitor did not short circuit.

These results demonstrate the complex interplay between electrical and mechanical subsystems. By incorporating real-world material properties (***aluminum plates, air as the dielectric, and stainless steel 302 for the spring***), the simulation provides a realistic representation of the system's behavior.

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